

Centered Weight Normalization in Accelerating Training of Deep Neural Networks

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1. Introduction

Optimization in Deep Model

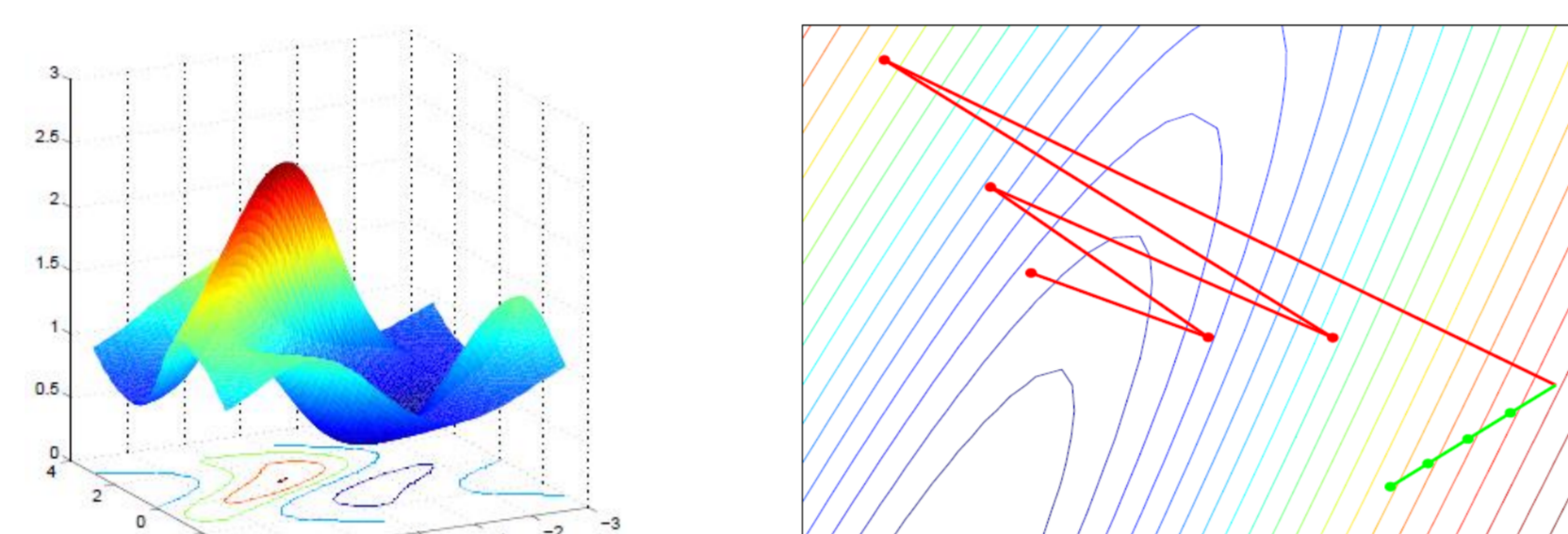
- Goal: $\theta^* = \arg \min_{\theta} \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \in D} [\mathcal{L}(\mathbf{y}, f(\mathbf{x}; \theta))]$
- Update Iteratively: $\theta^{(t+1)} \leftarrow \theta^{(t)} - \alpha^{(t)} \nabla^{(t)}$
- Challenge: Non-convex, ill conditioning

$$\begin{aligned} \mathbf{s}^l &= \mathbf{W}^l \mathbf{h}^{l-1} + \mathbf{b}^l \\ \mathbf{h}^l &= \varphi(\mathbf{s}^l) \end{aligned}$$

Stochastic gradient descent

- Gradient is averaged by the sampled examples

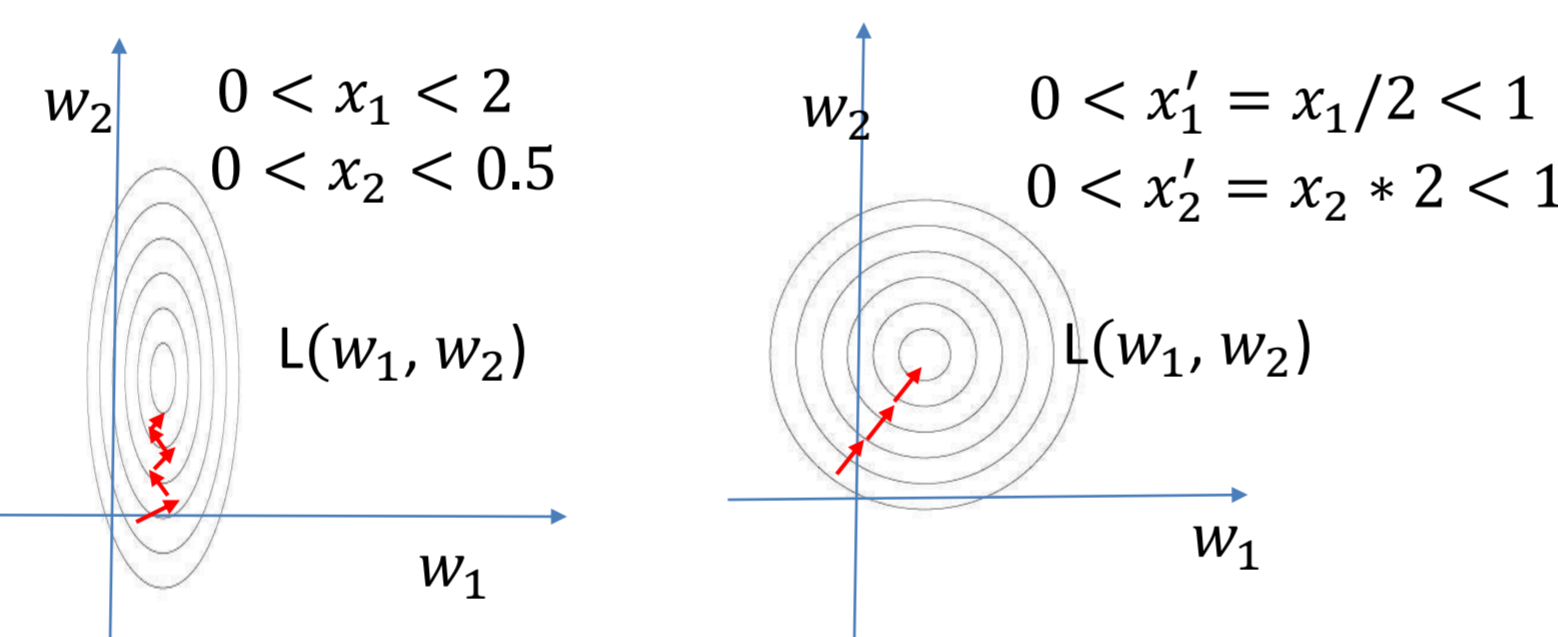
$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{1}{m} \sum_{i=1}^m \frac{\partial \mathcal{L}(\mathbf{y}_i, f(\mathbf{x}_i; \theta))}{\partial \theta}$$



Estimate curvature or scale

- Quadratic optimization: Newton, quasi-Newton, Natural Gradient
- Estimate the scale: AdaGrad, Rmsprop, Adam

$$\begin{aligned} y &= w_1 x_1 + w_2 x_2 + b \\ L &= (y - \hat{y})^2 \end{aligned}$$



Normalize input/activation

- Normalize explicitly: batch normalization
- Normalize implicitly (constrain weights): weight normalization

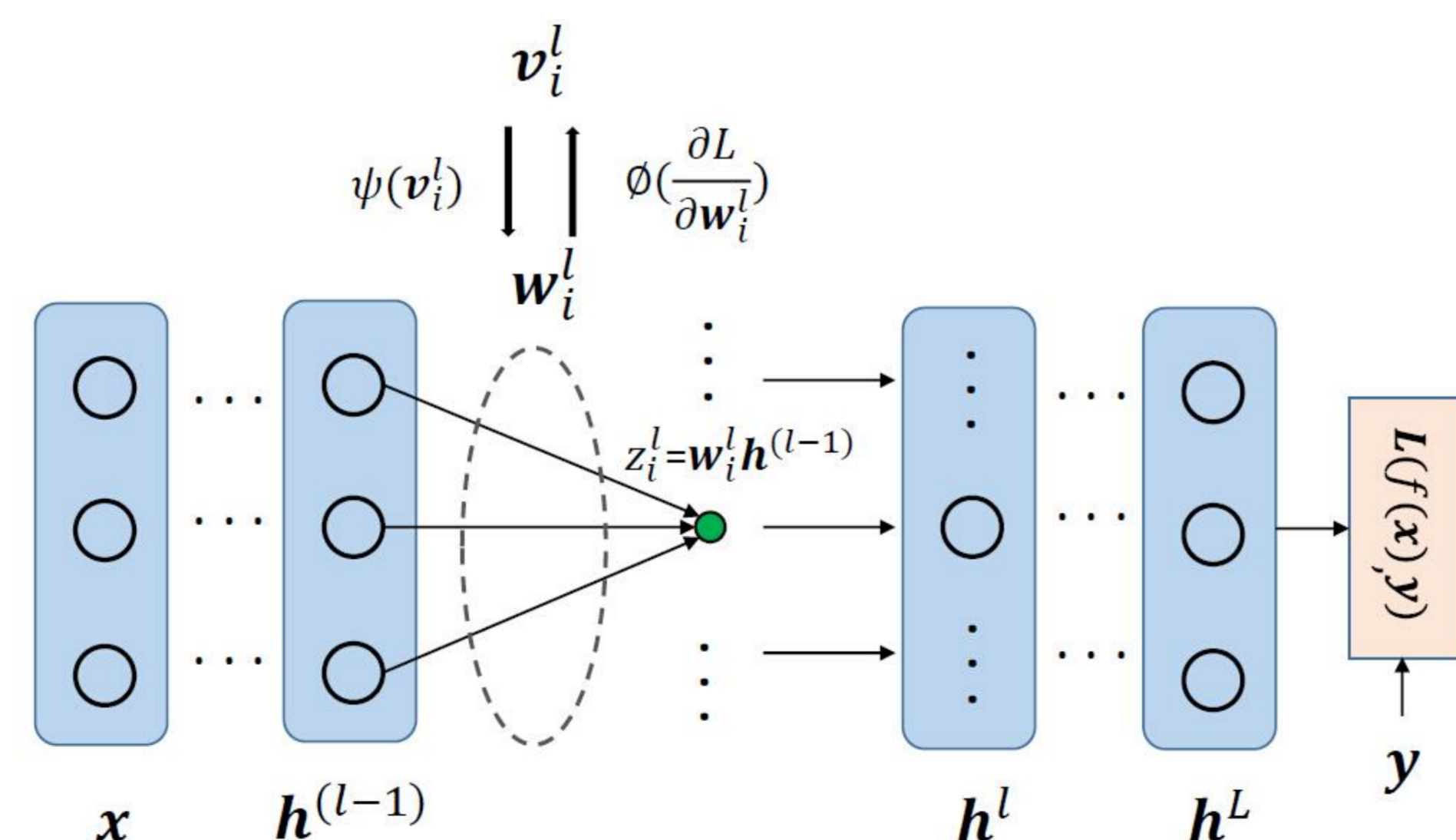
2. Motivation

Initialization methods

- Random, Xavier, MSRLnit: Zero mean, stable-variance
- Keep desired characters during training

Formulation

$$\begin{aligned} \theta^* &= \arg \min_{\theta} \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \in D} [\mathcal{L}(\mathbf{y}, f(\mathbf{x}; \theta))] \\ \text{s.t. } &\mathbf{w}^T \mathbf{1} = 0 \text{ and } \|\mathbf{w}\| = 1 \end{aligned}$$



3. Method

Solution by re-parameterization

- Proxy parameter \mathbf{v} :
$$\mathbf{w} = \frac{\mathbf{v} - \frac{1}{d} \mathbf{1} (\mathbf{1}^T \mathbf{v})}{\|\mathbf{v} - \frac{1}{d} \mathbf{1} (\mathbf{1}^T \mathbf{v})\|}$$
- Adjustable scale: $z = g \mathbf{w}^T \mathbf{h} + b$

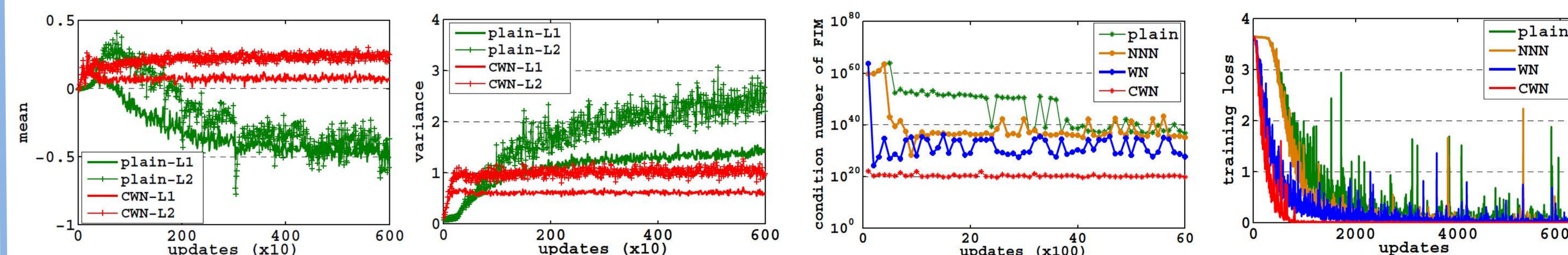
- Back-propagated Gradient

$$\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{1}{\|\hat{\mathbf{v}}\|} \left[\frac{\partial \mathcal{L}}{\partial \mathbf{w}} - \left(\frac{\partial \mathcal{L}}{\partial \mathbf{w}} \mathbf{w} \right) \mathbf{w}^T - \frac{1}{d} \left(\frac{\partial \mathcal{L}}{\partial \mathbf{w}} \mathbf{1} \right) \mathbf{1}^T \right]$$

Beneficial Properties

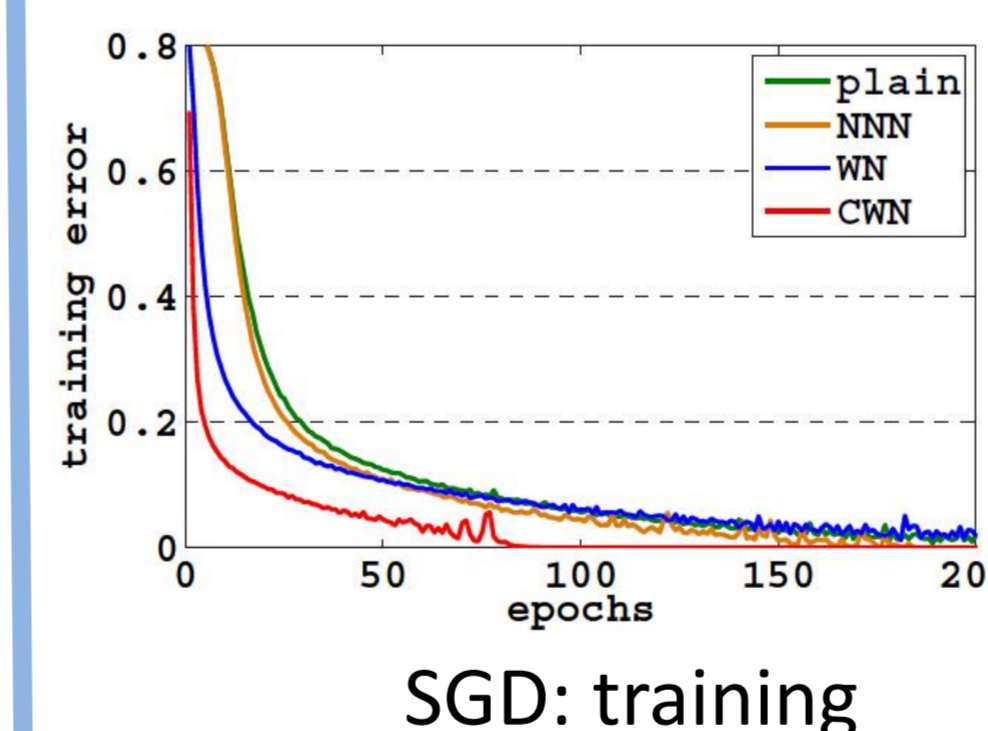
- Stabilize the distributions
- Better Conditioning of Hessian
$$\frac{\partial L}{\partial v} \cdot \mathbf{1} = \mathbf{0}$$

Proposition 1. Let $z = \mathbf{w}^T \mathbf{h}$, where $\mathbf{w}^T \mathbf{1} = 0$ and $\|\mathbf{w}\| = 1$. Assume \mathbf{h} has Gaussian distribution with the mean: $\mathbb{E}_{\mathbf{h}}[\mathbf{h}] = \mu \mathbf{1}$, and covariance matrix: $\text{cov}(\mathbf{h}) = \sigma^2 \mathbf{I}$, where $\mu \in \mathbb{R}$ and $\sigma^2 \in \mathbb{R}$. We have $\mathbb{E}_z[z] = 0$, $\text{var}(z) = \sigma^2$.

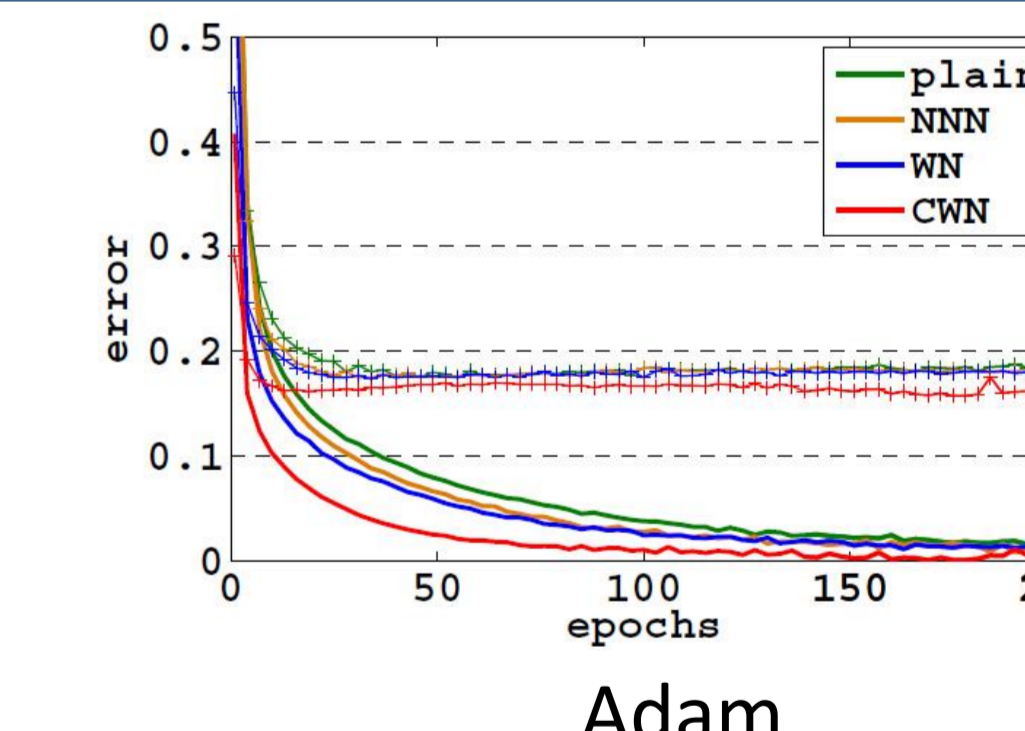
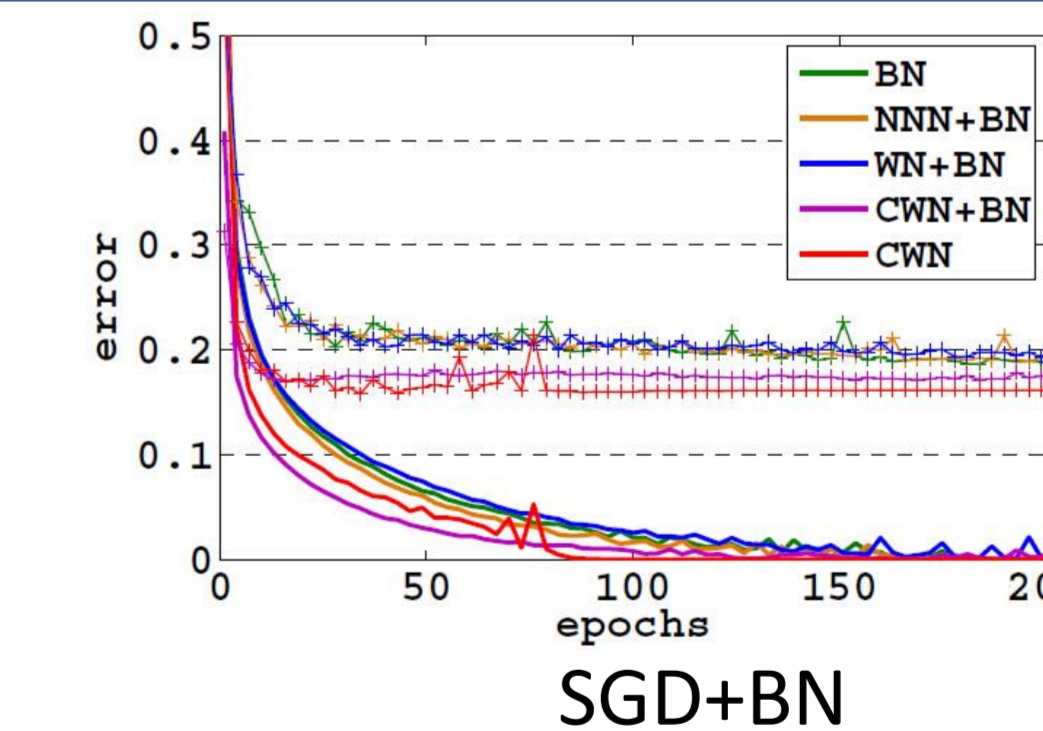
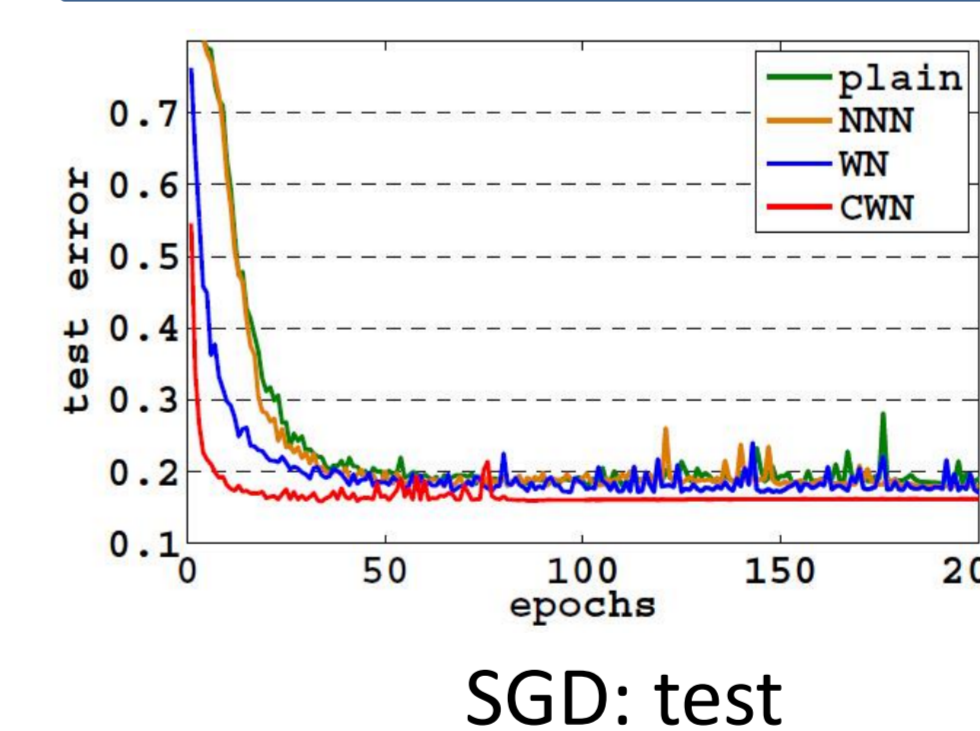


4. Experiments

MLP; SVHN



Simply replace Linear/Conv module with CWN module!



CNN architecture

- BN-Inception

	Cifar-10	Cifar-100
Plain	6.14 ± 0.04	25.52 ± 0.15
WN	6.18 ± 0.34	25.49 ± 0.35
WCBN	6.01 ± 0.16	24.45 ± 0.54

- 56 layers residual network

	Cifar-10	Cifar-100
Plain	7.34 ± 0.52	29.38 ± 0.14
WN	7.58 ± 0.40	29.85 ± 0.66
WCBN	6.85 ± 0.25	29.23 ± 0.14

- BN-Inception, ImageNet 2012

Methods	Top-1 error	Top-5 error
plain	30.78	11.14
WN	28.64	9.7
CWN	26.1	8.35

Code: <https://github.com/huangleiBuaa/CenteredWN>

